Charles Darwin University

Final Examination

Family Name	
Given Name/s	
Student Number	
Teaching Period	Semester 2, 2018

ENG476 – Control Engineering	DURATION
	Reading Time: 10 minutes
	Writing Time: 180 minutes

INSTRUCTIONS TO CANDIDATES

1. Answer all questions.
2. Note that questions ARE NOT of equal value.
3. Read ALL questions carefully.
4. Do not commence writing until instructed to do so.

EXAM CONDITIONS

<u>You may begin writing from the commencement of the examination session.</u> The reading time indicated above is provided as a guide only.

This is a RESTRICTED OPEN BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
Lecture Textbook/s (Unannotated)	1 x 20 Page Book

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Introduction to Questions 1 – 6: Start-Up Company "Cheap-Launch"

After graduating from CDU with a professional engineering qualification, you decide with three other graduates from CDU (another engineering graduate and two business graduates) to establish the start-up company "Cheap-Launch". The company aims to become a global disruptor of the space industry, by developing a very low cost methodology to launch small satellites. If successful, it would enable research organisations and universities to carry out independent investigations on the effects of global warming, significantly improving the understanding these processes and possibly contribute to mitigating strategies.

Your team decides that a reusable reverse landing rocket design (similar to Elon Musk's SpaceX space vehicle) is the preferred methodology to launch the satellites as this will significantly reduce operational cost.

A launch site for testing is selected in East Arnhem Land of the Northern Territory for its sparse population and closeness to the equator. Negotiations with the traditional owners have been successful and a lease for five years has been secured. Financial grants from the Northern Territory Government enable Cheap-Launch to establish appropriate facilities at the site, including a launch pad, and a control room in a concrete bunker.

You and your fellow engineering graduate realise that the rocket angle control system is critical for success. You focus your efforts on developing accurate models of the system and selecting economical, reliable and effective sensors, actuators and control hardware.

The rocket is modelled as mass m [kg] with an inertia J [kg m^2], controlled by a rocket force F [N] under and angle γ [rad], refer to Figure 1. Research of the literature indicates that control of the rocket is most difficult during take-off and landing (speed around 0), and you therefore focus your controller design on this situation. Furthermore, the thrust of the rocket engine F is in balance with the gravitational force at the launch site and is therefore assumed constant for analysis of the control system.

Although the control system needs to control two angles of the rocket thurster, you decide to design the control system as two separate, but identical, control loops, one for each angle. Each control system therefore consists of an angle sensor of the rocket (ϕ [rad]), and an angle sensor of the rocket engine (γ [rad]).

To save costs, you decide to use the gyroscopes of your old iPhones to measure the angle ϕ and to use cheap potentiometers to measure γ. As control computers you use the processor of your old iPhones.

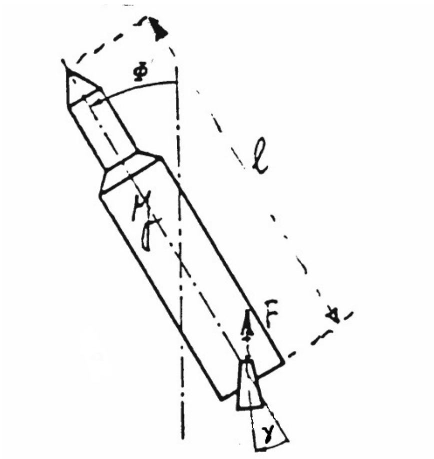

Figure 1, Schematic of rocket and variables

Using your knowledge of mechanics, you derive a theoretical Laplace domain model for the rocket as:

$$\frac{\phi(s)}{\gamma(s)} = G(s) = \frac{1}{2s^2}$$

The control system has as reference signal the desired angle of the rocket ϕ_r, assumed to be 0 at take-off and landing. The controller is modelled by the transfer function $G_c(s) = \frac{\gamma(s)}{e(s)}$, with $e(s) = \phi_r(s) - \phi(s)$.

Please note that all the following questions (Question 1 – 6) relate to the system above.

Question 1, Block Diagram (1 mark)

Draw a block diagram of the above closed loop control system. Label all elements of the block diagram appropriately.

Question 2, System with iPhone Gyroscope (3 marks)

You measure the frequency response of the open loop system including the iPhone gyroscope with $\gamma(j\omega)$ as input and $\phi(j\omega)$ as output, resulting in the Bode diagram of Figure 2. You suspect that the difference between the theoretical model and the measured response is due to the transfer function of the iPhone gyroscope.

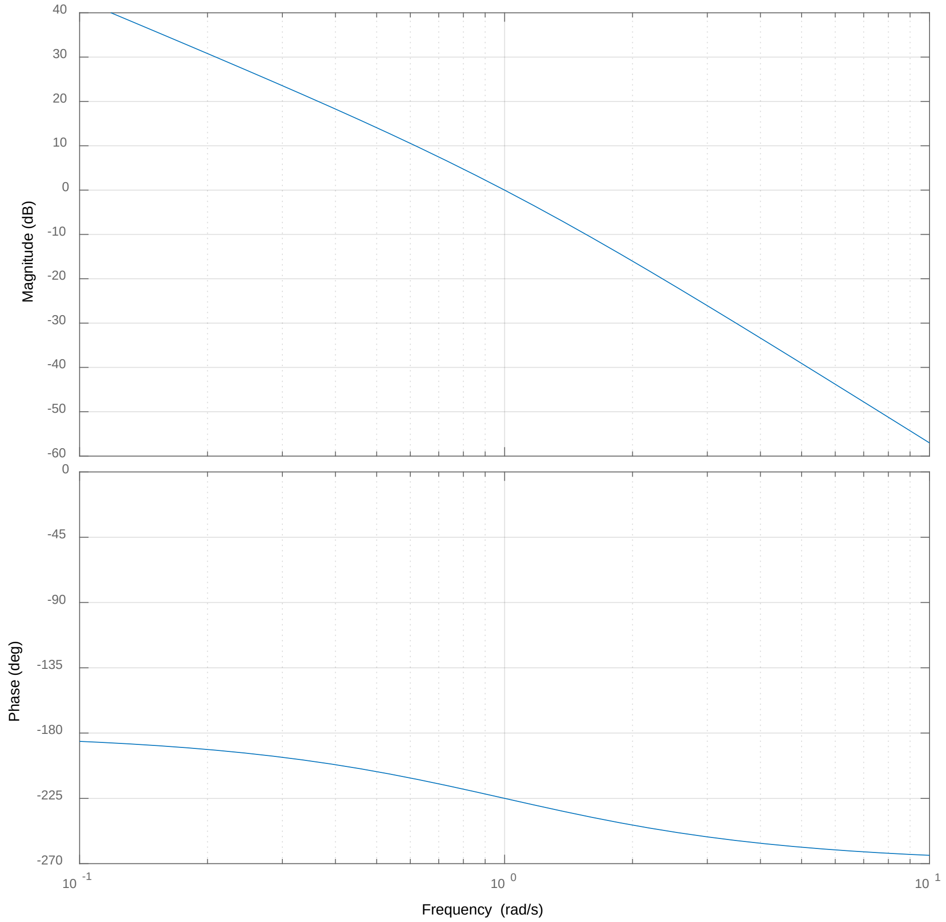

Figure 2, Bode diagram of measured frequency response with iPhone gyroscope.

Question 2.1 (1 mark)
Determine the magnitude and the phase of the iPhone gyroscope at ω = 1 [rad/s] and determine the transfer function, $G_{gyro}(s)$, of the gyroscope.

Question 2.2 (1 mark)
You decide to implement a PD controller, $G_c(s) = (s + 1)$, to control this system (the system of Figure 2). Determine the closed loop transfer function $G_{cl}(s)$.

Question 2.3 (1 mark)
Derive the equation for $\phi(t)$ when a unit step input is applied to the closed loop system.

Question 3, Improved System (3 marks)

You decide that the gyroscope of the old iPhone is not suitable for control of the rocket and instead you use a high quality gyroscope designed for professional drones. You measure again the frequency response of the open loop system with $\gamma(j\omega)$ as input and $\phi(j\omega)$ as output, resulting in the Bode diagram below in Figure 3.

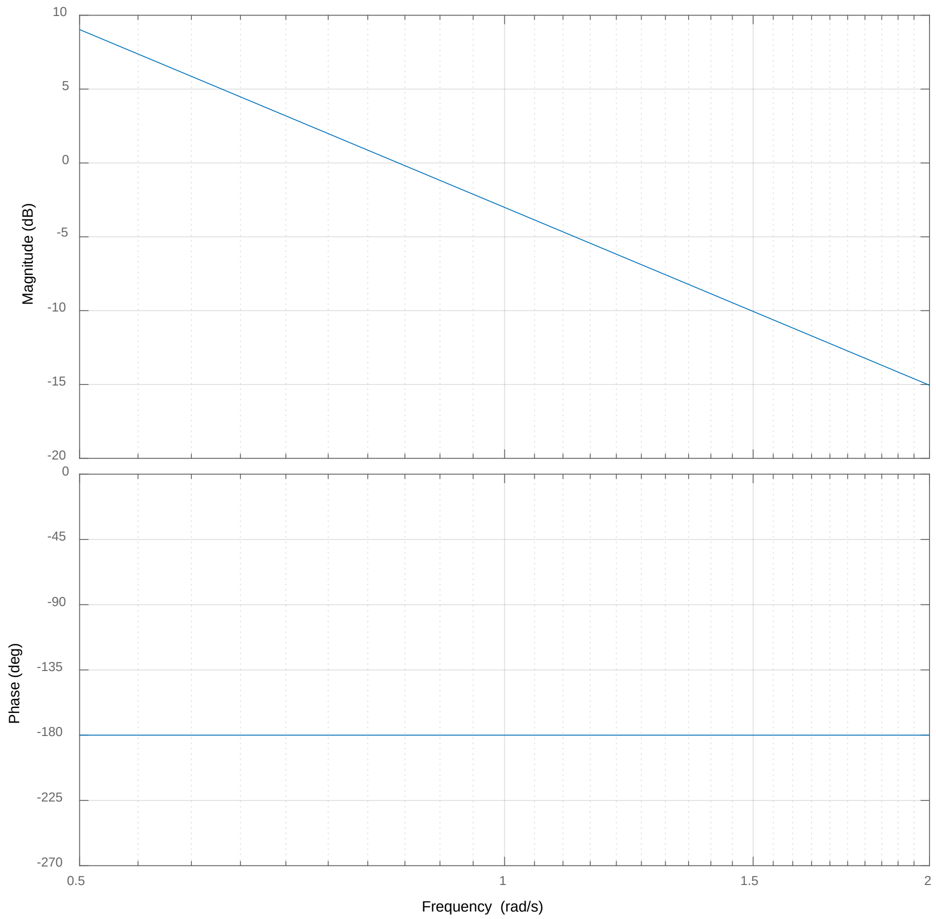

Figure 3, Bode diagram of improved open loop system.

Question 3.1 (1 mark)
Determine the transfer function $G_i(s)$ of the improved open loop system which uses the drone gyroscope.

Question 3.2 (1 mark)
Determine the magnitude [dB] and phase [deg] of the open loop system consisting of the PD controller, $G_c(s) = (s+1)$, and the improved system with high quality gyroscope for ω = 0, 0.5, 1, 2, ∞ [rad/s]. Sketch the polar plot of this system.

Question 3.3 (1 mark)
Determine the gain margin [dB] and the phase margin [deg] of the open loop system, consisting of the PD controller and the improved system with drone gyroscope.

Question 4, Compensator Design (4 marks) [1]

Instead of using the PD controller for closed loop control of the improved system, $G_i(s)$, you decide to use a compensator, applying unit negative feedback.

Specifications of the closed loop system are:

- Rise time = 1.21 [s]
- Overshoot = 16.3%

Question 4.1 (1 mark)

Determine the damping ratio, ζ, the undamped natural frequency, ω_n [rad/s], and the location of the desired dominant closed loop poles for the closed loop system.

Question 4.2 (1 mark)

Compute the angle deficiency at the location of the desired dominant closed loop poles.

Question 4.3 (1 mark)

Determine the location of the pole and the location of the zero of the compensator, $G_c(s)$, which will approximately achieve the above mentioned performance specifications. Compute T and α.

Question 4.4 (1 mark)

Using the magnitude condition for $G_c(s) \cdot G_i(s)$, compute K_c of the compensator.

[1] Only if you are unable to determine $G_i(s)$, use $\sqrt{2} \cdot G(s)$ mentioned above for this question instead.

Question 5, State Feedback (3 marks)

You also would like to test a state feedback controller. To do so, you differentiate the output of the high quality gyroscope, resulting in two states, $\phi(t)$ and $\dot{\phi}(t)$, available for feedback.

Question 5.1 (1 mark)[2]

Derive the state space model in controllable canonical form of the open loop system, $G_i(s)$, with $\gamma(t)$ as state space input $u(t)$, $[\phi(t) \;\; \dot{\phi}(t)]^T$ as state space state vector $x(t)$, and $\phi(t)$ as state space output $y(t)$.

Question 5.2 (1 mark)

Draw the block diagram of the closed loop state feedback controlled system, with $x_r(t) = [\phi_r(t) \quad 0]^T$ as input of the closed loop system, $\phi_r(t)$ being the reference angle for the rocket control system. Label all elements of the block diagram appropriately.

Question 5.3 (1 mark)

Using the "Direct Substitution Method", determine the state feedback matrix K of the state feedback controller $u(t) = -K \cdot u(t)$ which locates the closed loop poles at s = -2 and s = -3.

Question 6, Testing (1 mark)

You now proceed to test the various controllers in practice on the improved system (with the high quality gyroscope). During testing, the rocket hovers around its vertical axis just above the launch pad.

You note that the state feedback controlled system reacts quicker and with less major (low frequency) oscillations after wind gusts than the PD controlled or compensator controlled system. However, the system shakes continuously at frequencies well above 10 [rad/s], with or without wind gusts present.

Explain what two features of the control system may cause these high frequency oscillations when state feedback is applied. How would you change the system to try to reduce the high frequency oscillations?

[2] Only if you are unable to determine $G_i(s)$, use $\sqrt{2} \cdot G(s)$ mentioned above for this question instead.